MAC-CPTM Situations Project

Situation 06: Can You Always Cross Multiply?

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Prompt

This is one of several lessons in an algebra I unit on simplifying radical expressions. The teacher led students through several examples of how to simplify radical expressions when the radicands are expressed as fractions.

The class is in the middle of an example, for which the teacher has written the following on the whiteboard:

$$\sqrt{\frac{12}{81}} = \sqrt{\frac{3 \cdot 4}{3 \cdot 27}} = \sqrt{\frac{4}{27}} =$$

A student raises her hands and asks, "When we're doing this kind of problem, will it always be possible to cross multiply?"

Commentary

Mathematical Foci

Mathematical Focus 1

"Cross multiply" is a process that applies when working with proportions. The principle is that in the proportion, $\frac{a}{b} = \frac{c}{d}$, it is true that $a \cdot d = b \cdot c$. The student has recognized in the problem that $3 \cdot 27 = 81$, the denominator of the fraction in the left member of the equation, and $3 \cdot 4 = 12$, the denominator of the fraction in the right member of the equation where $\sqrt{\frac{12}{81}} = \sqrt{\frac{3 \cdot 4}{3 \cdot 27}}$. We do not have a proportion in this problem. So, it would be better not to think about cross multiplying in this type of problem.

Mathematics used for Mathematical Focus 1

- Be able to distinguish an equation that is a proportion from an equation whose elements contain fractional forms.
- Recognize "cross multiply" as a term associated with the principle, $\frac{a}{b} = \frac{c}{d}$ $(b, d \neq 0) \Rightarrow a \cdot d = b \cdot c$.

Mathematical Focus 2

Assume what the student means is that it will always be possible to find factors to use in the second radicand that would produce the numerator and denominator of the original radicand when multiplied in the order shown here:

$$\sqrt{\frac{12}{81}} = \sqrt{\frac{3 \cdot 4}{3 \cdot 27}}$$

$$\sqrt{\frac{100}{225}} = \sqrt{\frac{25 \cdot 4}{25 \cdot 9}}$$

Looking at several more examples suggests that this idea always works when the numerator and denominator of the original radicand have a common factor. The following counterexamples show that the idea does not apply when there is no common factor other than 1:

$\sqrt{\frac{14}{15}} = \sqrt{\frac{2 \cdot 7}{3 \cdot 5}}$ yields 10 and 21, not 14 and 15

$\sqrt{\frac{14}{15}} = \sqrt{\frac{2 \cdot 7}{5 \cdot 3}}$ yields 6 and 35, not 14 and 15

$\sqrt{\frac{14}{15}} = \sqrt{\frac{7 \cdot 2}{3 \cdot 5}}$ yields 35 and 6, not 14 and 15

$\sqrt{\frac{14}{15}} = \sqrt{\frac{7 \cdot 2}{5 \cdot 3}}$ yields 21 and 10, not 14 and 15

Mathematics used for Mathematical Focus 2

- See how the products may be obtained in a manner that is similar to the usual "cross multiply" pattern.
- Know that it is important to look at cases when there are and are not common factors of denominators and numerators.
- Know that a common factor of 1 produces a different type of case.

Mathematical Focus 3

There is something about problems like this that involve a sense of "cross multiply." Each problem of this type starts with a radicand that is of the form $\frac{p}{q}$, where *p* and *q* are non-negative integers and $q \neq 0$. Both *p* and *q* may be factored over the integers as $p=kn$ and $q=km$, where *k* is a positive integer. So, we have

$$\sqrt{\frac{p}{q}} = \sqrt{\frac{k \cdot n}{k \cdot m}}$$

The "cross multiply" notion yields *km* and *kn*, which are *q* and *p*, respectively. So, yes, this view of cross multiply will always work when the original radicand is a positive fraction.

Mathematics used for Mathematical Focus 3

- See how the products may be obtained in a manner that is similar to the usual "cross multiply" pattern.
- Know that it is important to look at cases when there are and are not common factors of denominators and numerators.
- Know that a common factor of 1 produces a different type of case.

Mathematical Focus 4

Under what conditions does the student's observation work? That is, for equations such that $\sqrt{\frac{p}{q}} = \sqrt{\frac{a_1 \cdot a_2}{b_1 \cdot b_2}}$ under what conditions is it true that ($a_1 \cdot b_2 = p$ and $a_2 \cdot b_1 = q$ $a_1 \cdot b_2 = p$) or ($a_1 \cdot b_2 = q$ and $a_2 \cdot b_1 = p$) or ($a_1 \cdot b_1 = q$ and $a_2 \cdot b_2 = p$) or ($a_2 \cdot b_2 = q$ and $a_1 \cdot b_1 = p$). We can show that this is true only when $a_1 = b_1$ or $a_1 = b_2$ or $a_2 = b_1$ or $a_2 = b_2$

Mathematics used for Mathematical Focus 4

- Generate an abstract case of an algebraic relationship proposed in concrete form.

References

None